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Analytical Solution of the Effect of MHD Inclination and Unsteady Heat Transfer in a Laminar, Transition and Turbulent Flow of a Basic Gaseous Micro-Flow past a Vertically Moving Oscillating Plate

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Abstract: *The effect of MHD inclination and heat transfer on an unsteady natural convection of a basic gaseous micro-flow over an infinite vertically moving oscillating plate is investigated in this paper analytically using air as a working fluid. Three basic flow cases are considered in the flow regime: the Laminar flow, the transition flow and the turbulent flow. It is assumed that the fluid flow over the right surface of the oscillating plate with a constant velocity. The governing partial differential equations have been made dimensionless using a transformation technique into a system of ordinary differential equations, which are then solved analytically. The effect of MHD and its inclination on the dimensionless velocity is illustrated graphically and analyzed in greater detail. In addition, numerical results of skin friction and heat transfer rate are also presented.*

Keywords: MHD Inclination, Reynolds number, Moving Oscillating Vertical Plate, Hartman number, Grashof number.

I. Introduction

The effect of natural convection MHD of basic gaseous micro-flow over a vertically moving oscillating plate has received considerable attention because of its many practical applications in engineering systems such as MHD devices, combustion chamber, cooling of nuclear reactors, etc. Magnetic fluid interaction with fluids consequently modifies flow properties like velocity, skin friction co-efficient and heat transfer in which the main cause strongly depends on the orientation of the magnetic field. Many recently published literature articles concerning MHD natural convection flows over a flat plate/moving plate have been found [1, 2]. Chamkha and Aly [3] studied the effect of heat generation and absorption on MHD free convection flow of nanofluid past a vertical plate. More to this, Stamenkovic et al [4] investigates MHD flow of two immiscible and electrically conducting fluids between isothermal insulated moving plates in the presence of applied electric and magnetic field. They concluded that decrease in magnetic field inclination angle flattens out the velocity and temperature profiles. Very recently, Navneet et al [5] carried out an investigation on the effect of incline magnetic field on unsteady flow past on moving vertical plate with variable temperature. He also concluded from his work that an increase in angle of inclination of magnetic field causes a decrease in flow velocity. Radiation and chemical reaction effects on MHD Casson fluid flow past an oscillating vertical plate embedded in porous medium was analyzed by Kataria et al [6]. They concluded that velocity increases and temperature decreases with increase in thermal radiation. Tripathy et al [7] investigated chemical reaction effect on MHD free convective surface over a moving vertical plate through porous medium. They concluded that increase in local magnetic parameter Ha results in a decrease in velocity. Sandeep and Sugunamma [8] carried out an investigation on radiation and inclined magnetic effects on unsteady hydromagnetic free convection flow past an impulsively moving vertical plate in a porous medium, where they concluded that magnetic field decreases the velocity of the fluid.

However, to the best of authors' knowledge no attempt has been made to study the effect of MHD inclination and unsteady heat transfer in a laminar, transition and turbulent flow of a basic gaseous micro-flow past a vertically moving oscillating plate. Hence, the problem is therefore investigated.

II. Formulation of the problem

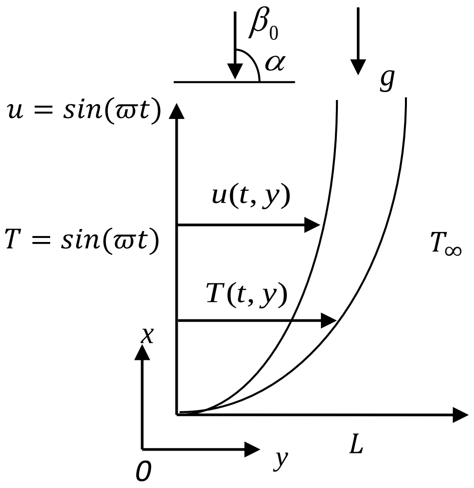


Fig. 1: Schematic of the problem

Consider an unsteady natural convection, fully developed incompressible flow of a basic micro flow gas. The fluid flows over the right surface of an infinite vertical plate oscillating in its own plane with a constant velocity $u = \sin(\bar{\omega} t)$ in a stationary fluid domain as seen in Fig 1 above. Slip velocity and thermal slip are considered at the boundary of the oscillating plate. The governing equations with the boundary conditions can be written in dimensional form as:

Governing Equations:

$$\rho\frac{\partial u}{\partial t} = \mu\frac{\partial^2 u}{\partial y^2} + \rho g\beta(T - T_\infty) - \sigma\beta_o{}^2 u\sin^2\alpha \qquad 1$$

$$\rho C_p\frac{\partial T}{\partial t} = k\frac{\partial^2 T}{\partial y^2} \qquad 2$$

Boundary Conditions:

Velocity and Thermal slip are assumed at the wall and in contact with a stationary fluid at free stream.

$$u(t,\infty) = 0 \qquad 3$$

$$u(t,0) - \sin(\varpi t) = -\frac{2-\sigma_v}{\sigma_v}\lambda\left.\frac{\partial u}{\partial y}\right|_{y=0} \qquad 4$$

$$\frac{\partial T}{\partial y}(t,\infty) = 0 \qquad 5$$

$$T(t,0) - \sin(\varpi t) = -\frac{2-\sigma_T}{\sigma_T}\frac{\lambda}{\Pr}\frac{2\gamma}{1+\gamma}\left.\frac{\partial T}{\partial y}\right|_{y=0} \qquad 6$$

We now introduce the following dimensionless variables:

$$Y = \frac{y}{L},\ \tau = \frac{t}{t_r},\ \omega = \frac{\varpi}{\omega_r},\ Kn = \frac{\lambda}{L},\ \Pr = \frac{\upsilon}{\alpha},\ \theta = \frac{T - T_\infty}{T_w - T_\infty}, \qquad 7$$

$$Ha^2 = \frac{\sigma\beta_o^2 L^2}{\mu},\ \mathrm{Re} = \frac{u_o L}{\upsilon},\ Gr_L = \frac{g\beta L^2(T_w - T_\infty)}{\upsilon u_o},\ U = \frac{u}{u_o} \qquad 8$$

Substituting Eq. (7) and Eq. (8) into Eq. (1-6), we obtain:

$$\frac{\partial U}{\partial\tau} = \frac{\partial^2 U}{\partial Y^2} + \frac{Gr_L}{\mathrm{Re}}\theta - Ha^2 U\sin^2\alpha \qquad 9$$

$$\frac{\partial\theta}{\partial\tau} = \frac{1}{\Pr}\frac{\partial^2\theta}{\partial Y^2} \qquad 10$$

$$U(\tau,\infty) = 0,\ U(\tau,0) - \sin(\omega\tau) = -\frac{2-\sigma_v}{\sigma_v}Kn\left.\frac{\partial U}{\partial Y}\right|_{Y=0}$$

and 11

$$\frac{\partial\theta}{\partial Y}(\tau,\infty) = 0,$$

$$\theta(\tau,0) - \sin(\omega\tau) = -\frac{2-\sigma_T}{\sigma_T}\frac{Kn}{\Pr}\frac{2\gamma}{1+\gamma}\left.\frac{\partial\theta}{\partial Y}\right|_{Y=0} \qquad 12$$

Where α is the angle between β_0 and $u(t,y)$ for $0 \le \alpha \le \pi$ $\Pr = \frac{\upsilon}{\alpha}$ is the Prandtl number, $\mathrm{Re} = \frac{u_0 L}{\upsilon}$ is the Reynolds number, $\mathrm{Gr_L} = \frac{g\beta L^2(T_w - T_\infty)}{\upsilon u_0}$ is the Grashof number and $\mathrm{Ha} = \sqrt{\frac{\sigma}{\mu}}\beta_0 L$ is the Hartman number.

This type of model can be used in electromagnetic flow enhancement and heating of materials. Eq. (9) and (10) will give an exact solution for the problem. Since the flow is assumed to be hydro dynamically fully developed, change in velocity and temperature with respect to x is zero. An exact solution for the problem is possible by assuming the following complex solution:

$$U(\tau,Y) = \mathrm{Im}\{\exp(i\omega\tau)V(Y)\} \text{ and}$$
$$\theta(\tau,Y) = \mathrm{Im}\{\exp(i\omega\tau)W(Y)\} \qquad 13$$

Where Im represents the imaginary part of the complex solution and $i = \sqrt{-1}$. By substituting Eq. 13 into the governing equations and boundary conditions, the following transformed ODEs are obtained:

$$W(Y) = \frac{1}{1-n}\exp(-Y\sqrt{i\omega\Pr}) \qquad 14$$

$$V(Y) = \left(\frac{1}{1-m} + \frac{Gr_L(b-1)}{\mathrm{Re}(1-m)(n-1)(i\omega\Pr - i\omega - M^2)}\right) \times\left(\exp\left(-Y\sqrt{i\omega + M^2}\right)\right) + \frac{Gr_L\exp\left(-Y\sqrt{i\omega\Pr}\right)}{\mathrm{Re}(n-1)(i\omega\Pr - i\omega - M^2)} \qquad 15$$

$$W(Y) = \frac{1}{1-n}\exp(-Y\sqrt{i\omega\Pr}) \qquad 16$$

$$U(\tau,Y)=\mathrm{Im}\left\{\begin{array}{l}\exp(i\omega\tau)\left(\dfrac{1}{1-m}+\dfrac{Gr_L(b-1)}{\mathrm{Re}(1-m)(n-1)(i\omega\mathrm{Pr}-i\omega-M^2)}\right)\\ \times\left(\exp\left(-Y\sqrt{i\omega+M^2}\right)\right)+\dfrac{Gr_L\exp\left(-Y\sqrt{i\omega\mathrm{Pr}}\right)}{\mathrm{Re}(n-1)(i\omega\mathrm{Pr}-i\omega-M^2)}\end{array}\right\} \qquad 17$$

$$\theta(\tau,Y)=\mathrm{Im}\left\{\exp(i\omega\tau)\left(\frac{1}{1-n}\exp(-Y\sqrt{i\omega\mathrm{Pr}})\right)\right\} \qquad 18$$

Where

$m=\dfrac{2-\sigma_v}{\sigma_v}Kn\sqrt{i\omega+M^2}$, $n=\dfrac{2-\sigma_T}{\sigma_T}\dfrac{Kn}{\mathrm{Pr}}\dfrac{2\gamma}{1+\gamma}\sqrt{i\omega\mathrm{Pr}}$,

$b=\dfrac{2-\sigma_v}{\sigma_v}Kn\sqrt{i\omega\mathrm{Pr}}$ and $M^2=Ha^2\sin^2\alpha$

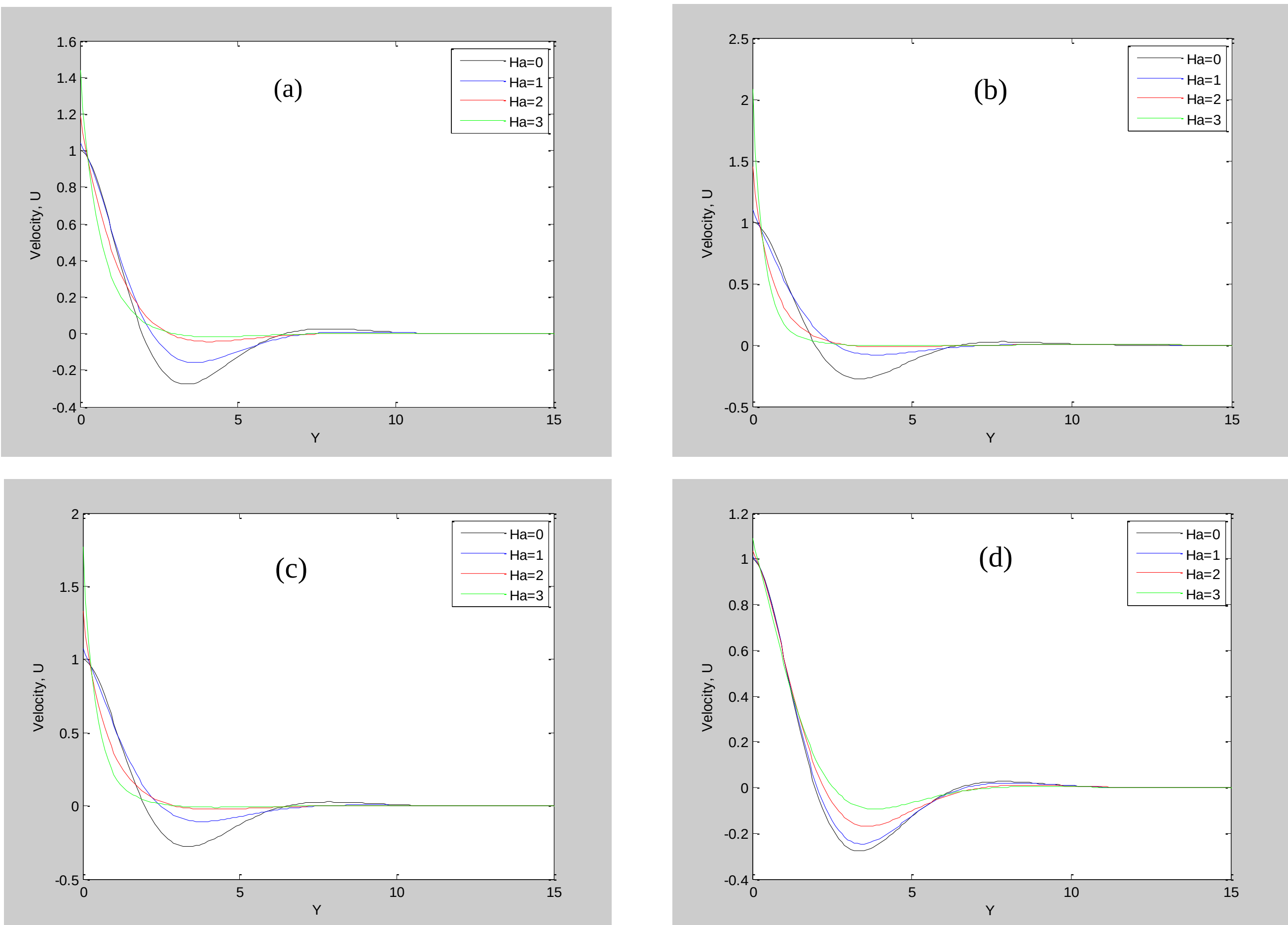
Physical quantities of interest in this study include the skin friction τ_w and the wall heat flux q_w which are defined as

$$\tau_w=\left.\frac{\partial U}{\partial Y}\right|_{Y=0}=\mathrm{Im}\left\{\exp(i\omega\tau)\left[\begin{array}{l}\left(-\sqrt{i\omega+M^2}\right)\times\\ \left(\dfrac{1}{1-m}+\dfrac{Gr_L(b-1)}{\mathrm{Re}(1-m)(n-1)(i\omega\mathrm{Pr}-i\omega-M^2)}\right)\\ +\dfrac{\left(-\sqrt{i\omega\mathrm{Pr}}\right)Gr_L}{\mathrm{Re}(n-1)(i\omega\mathrm{Pr}-i\omega-M^2)}\end{array}\right]\right\} \qquad 19$$

$$q_w=\left.\frac{\partial\theta}{\partial Y}\right|_{Y=0}=\mathrm{Im}\left\{\exp(i\omega\tau)\left(\frac{\sqrt{i\omega\mathrm{Pr}}}{n-1}\right)\right\} \qquad 20$$

III. Results and Discussion

A parametric study is performed to illustrate the effect of MHD inclination on the flow properties of a moving oscillating vertical plate using air as a working fluid. This effect of MHD inclination is investigated analytically and presented graphically in Fig. (2-4), the choice of values of flow dimensionless parameters were based on values reported in literature. Prandtl number Pr of 0.71 is chosen for air, specific heat ratio γ, of 1.4 for basic micro-gas. While Reynolds number Re, Knudsen number Kn and Grashof number Gr_L were chosen based on the types of flow considered. Re of 10^3, 3×10^3 and 12×10^3 were used throughout the computation for laminar, transition and turbulent respectively. For Grashof number, positive values were utilized which corresponds to cooling effect with respect to engineering application. Gr_L of 10^3, 10^5 and 10^8 were used respectively for laminar, transition and turbulent. While Kn of 10^{-1}, 10^0 and 10^{-1} were also used for the same flow.



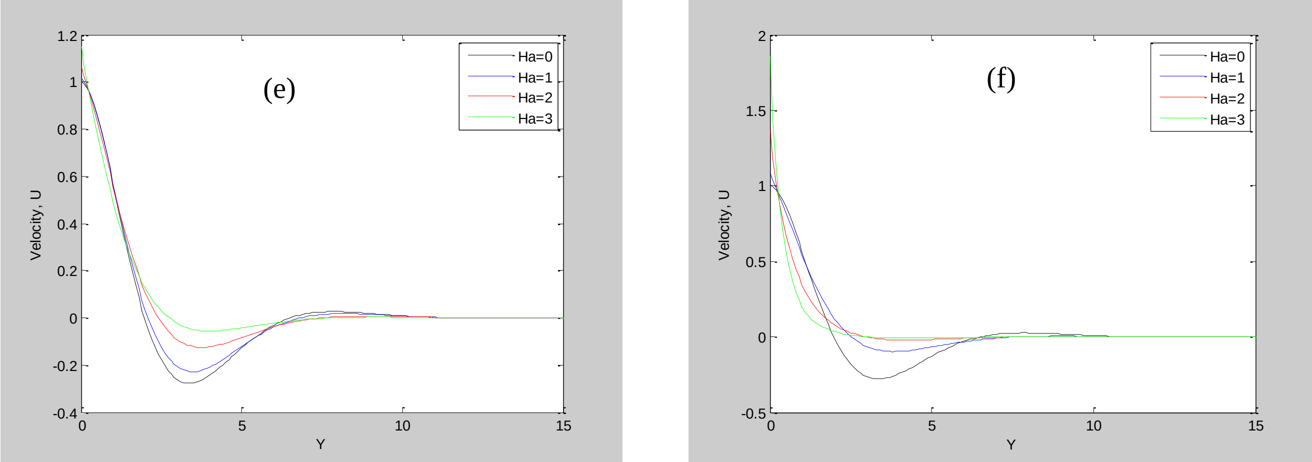
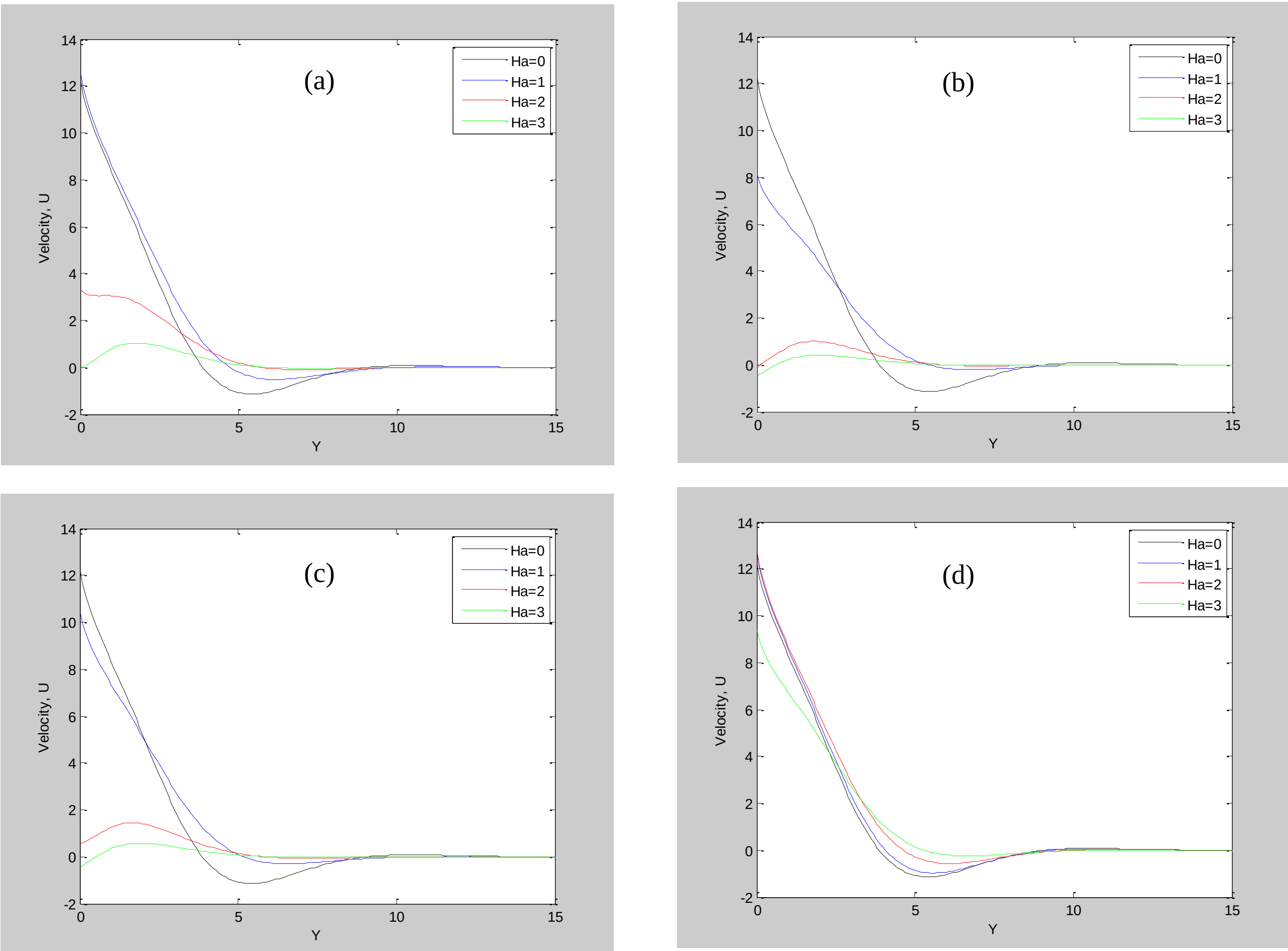


Fig. 2: Laminar flow at $\mathrm{Re}=10^3, Gr_L=10^3$, $\mathrm{Pr}=0.71, \omega\tau=\pi/2$, $Kn=10^{-1}$, $\gamma=1.4$, $\omega=1$ (a) $\alpha=15^o$, (b) $\alpha=30^o$, $(c)\alpha=45^o$, $(d)\alpha=60^o$, $(e)\alpha=75^o$, $(f)\alpha=90^o$

Fig. 2 (a-f) above shows the effect of MHD inclination at different angles in a laminar flow. The result shows that magnetic field in general increases the velocity of air in a sinusoidal manner. An MHD inclination increase wall velocity in this flow, with an optimum velocity of about 2.05 found at an inclination of 30^o. An inclination of 90^o gives a wall velocity close to the optimum, as it gives a velocity of about 1.85 at a magnetic field of $Ha=3$. A wall velocity of about 1.75 is recorded for an inclination of 45^o and the least is recorded with 60^o inclination.



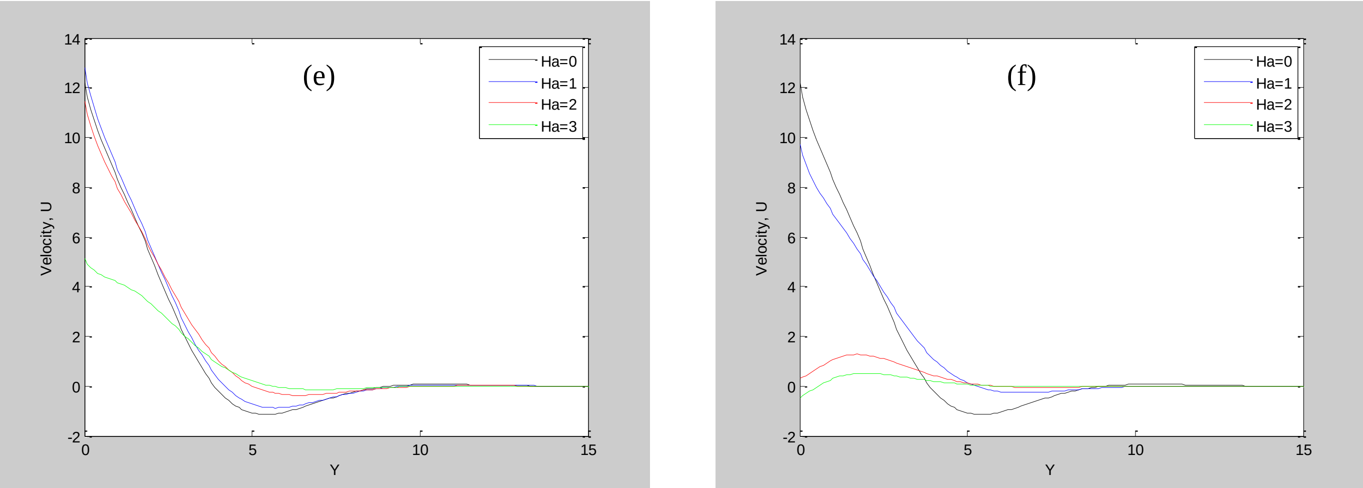
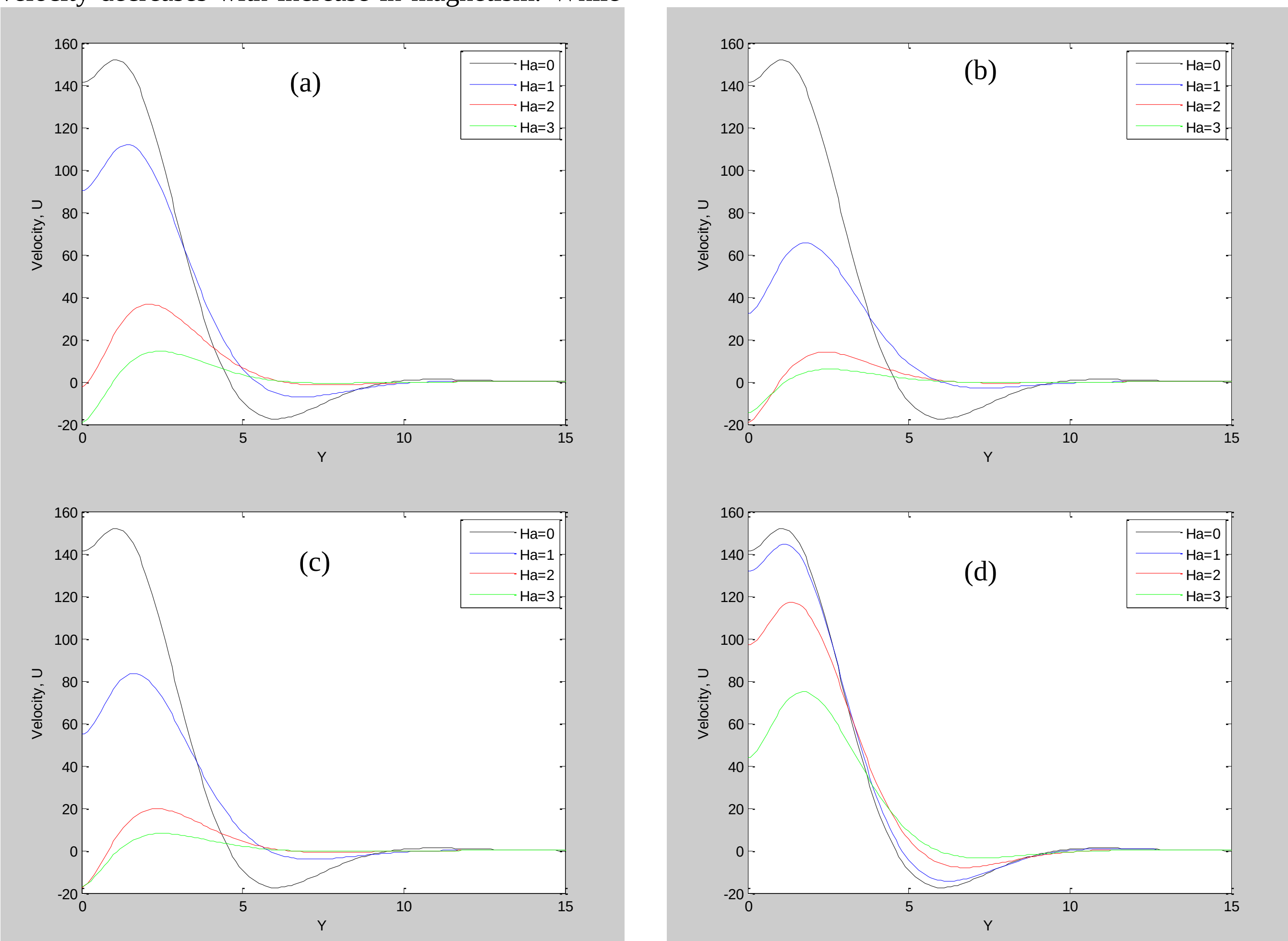
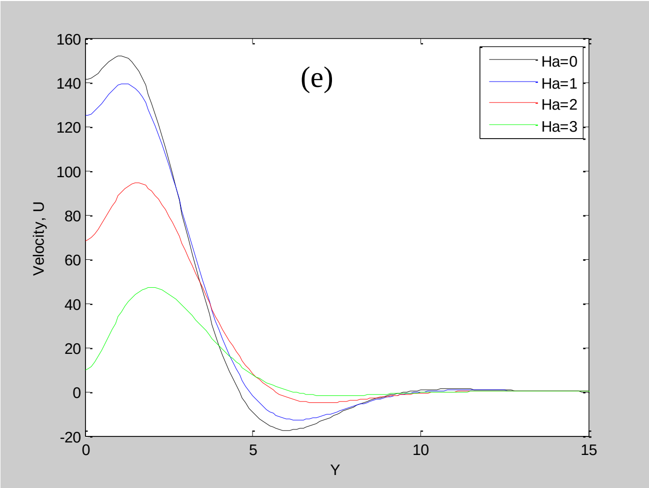


Fig. 3: Transition flow at $\mathrm{Re} = 3\times10^3$, $Gr_L = 10^5$, $\mathrm{Pr} = 0.71$, $\omega\tau = \pi/2$, $Kn = 10^0$, $\gamma = 1.4$, $\omega = 1$ (a) $\alpha = 15^o$, $(b)\,\alpha = 30^o$, $(c)\,\alpha = 45^o$, $(d)\,\alpha = 60^o$, $(e)\,\alpha = 75^o$, $(f)\,\alpha = 90^o$.

Fig. 3 (a-f) represents MHD inclination effect at different angles in a transition flow. By inspecting these figures, it is observed across the flow field that a decrease in magnetism brings about a decrease in fluid velocity away from the wall, this is to say, magnetism varies proportionally with the fluid velocity. At an inclination of 30^o, 45^o and 90^o, wall velocity decreases with increase in magnetism. While at 15^o and 75^o, wall velocity increases with increase in magnetism from $Ha = 0$ to 1, then decreases with increase in magnetism. With 60^o inclination, wall velocity increases with increase in magnetism from $Ha = 0$ to $Ha = 0$ and then decreases with $Ha = 3$ and above.





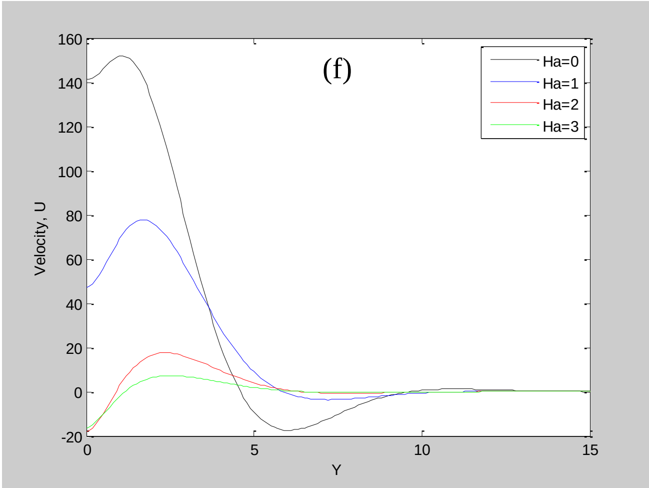


Fig. 4: Turbulent flow at $\text{Re}=12\times10^{3}$, $Gr_L=10^{8}, \text{Pr}=0.71, \omega\tau=\pi/2, Kn=10^{1}, \gamma=1.4, \omega=1$ $(a)\ \alpha=15^{o}$, $(b)\alpha=30^{o}$, $(c)\alpha=45^{o}$, $(d)\alpha=60^{o}$, $(e)\alpha=75^{o}$, $(f)\alpha=90^{o}$

As seen also in turbulence flow profile from Fig. 4 (a-f), an increase in magnetism is accompanied by decrease in wall velocity of the fluid, but across the flow and far from the wall, an increase in magnetism brings about an increase in fluid velocity in a sinusoidal manner. Throughout the flow, magnetism at zero isn't affected by α^{o} since the magnetism is absent and therefore it's a constant. An MHD inclination of 60^{o} gives the optimum wall velocities in this case at a magnetism of $Ha=1,2$ and 3 respectively. The order of decrease in wall velocity with respect to MHD inclination in a turbulent flow is $60^{o}, 75^{o}, 15^{o}, 45^{o}, 90^{o}$ and 30^{o}.

Table 1: Exact values of Skin friction when $\omega=1$ and $\omega\tau=\pi/2$

Flow Case	Re	Gr_L	Ha	α^o	Pr	τ_w
Laminar	1x10^3	10^3	1	15	0.71	-0.2201
				30		-0.5613
				45		-0.4015
				60		-0.0444
				75		-0.0727
				90		-0.4486
Transition	3x10^3	10^5	1	15	0.71	-6.2171
				30		-3.8115
				45		-5.0626
				60		-6.2625
				75		-6.3579
				90		-4.6923
Turbulent	12x10^3	10^8	1	15	0.71	-4.8092
				30		-1.6715
				45		-2.8936
				60		-7.0408
				75		-6.6836
				90		-2.4900

Table 1 illustrates the values of skin friction for different angles of MHD inclination in the system. It can be seen in the table that skin friction is proportional to the wall velocity. Maximum skin friction is attained at optimum wall velocity which corresponds to MHD inclination of $30°, 75°$ and $60°$ for Laminar, Transition and Turbulent respectively. Negative sign of skin friction implies that a drag force is exerted by the fluid on the opposite side of the plate. It is observed from Table 2 below that heat transfer rate increases periodically with periodic time, with $\omega\tau = \frac{\pi}{2}$ corresponding to $\frac{3\pi}{2}$ and π corresponding to 2π approximately. Positive sign implies upper amplitude of heat transfer while negative sign implies the opposite.

Table 2: Exact values of wall heat flux (with periodic time, $\omega\tau$) when $\omega = 1, \sigma_T = 0.7, Pr = 0.71$ & $\gamma = 1.4$

$\omega\tau$	q_w		
	Laminar	**Transition**	**Turbulent**
$\pi/2$	-0.5392	0.3952	0.0337
π	0.8489	0.1494	0.0009
$3\pi/2$	0.5381	-0.3954	-0.0337
2π	-0.8495	-0.1489	-0.0009

IV. Concluding Remarks

In this paper, the effect of MHD inclination in a laminar, transition and turbulent flow of a basic gaseous micro-flow over a vertically moving oscillating plate was studied. The following conclusions out of this study were drawn:

1. Magnetic field increases the velocity of a basic micro-gas in a laminar flow, while MHD inclination increases wall velocity with an optimum velocity found at an inclination of 30^{o}.
2. Across the flow field in a transition flow, decrease in magnetism brings about decrease in fluid velocity. Wall velocity decreases with increase in magnetism at

an angle of inclination of 30^o, 45^o and 90^o, while at 30^o, 75^o and 60^o, it rises with increasing magnetism.

3. In a turbulent flow, increasing magnetism is accompanied by decrease in wall velocity of the fluid, and optimum velocity is achieved at an inclination of 60^o.
4. Skin friction is proportionate to the wall velocity.
5. Heat transfer rate increases periodically with periodic time. Positive sign implies upper amplitude of heat transfer while negative sign implies the opposite.

Nomenclature

C_p	-specific heat
k	-thermal conductivity
Kn	-Knudsen number $(=\lambda / L)$
L	-reference length
Pr	-Prandtl number $(=\upsilon / \alpha)$
T	-temperature
T_∞	-ambient temperature
T_W	-wall temperature
t	-time
t_r	-reference time $(=L^2 / \upsilon)$
ϖt	-periodic time
u	-axial velocity (in x-direction)
u_o	-velocity of the moving wall
V	-complex solution function for velocity
W	-complex solution function for temperature
x, y	-Cartesian co-ordinate
g	-gravitational acceleration

Greek symbols

α	-thermal diffusivity
β_0	-magnetic field strength
γ	-specific heat ratio (C_p / C_υ)
λ	-Mean-free-path length
ρ	-density
μ	-dynamic viscosity
υ	-kinematic viscosity
θ	-dimensionless temperature $(=T - T_\infty / T_W - T_\infty)$
β	-thermal expansion coefficient
σ_T	-thermal accommodation coefficient
σ_υ	-tangential momentum accommodation coefficient
ϖ	-frequency
ω	-dimensionless frequency (ϖ / ω_o)
ω_o	-dimensionless frequency $(=\upsilon / \varpi L^2)$
τ	-dimensionless time

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